Leibniz's Possible Worlds

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Contents

1. Leibniz's Motivation

- It has been said that one of the main motivations for Leibniz to establish the doctrine of possible worlds is to avoid the mistakes which had been committed by Spinoza who thought that everything is determined and nothing happens contingently.

- Leibniz's theory of complete concepts together with his principle of 'praedicatum inest subjecto' has invoked fierce debates over whether the predicate 'being the conqueror of Darius and Porus' is already contained in the subject 'Alexander the Great'.

Quotation

- *Discourse On Metaphysics*:
- 'the nature of an individual substance or of a complete being is to have a notion so complete that it is sufficient to contain and to allow us to deduce from it all the predicates of the subject to which this notion is attributed.'

- Moreover, though 'being a king' is not determinate enough to constitute an individual, God 'sees in it at the same time the basis and reason for all the predicates which can be said truly of him'.

2. Complete Concepts and Possible Individuals

- If the complete concept is so complete that everything that can happen to an individual substance or is true of the subject which represents that individual substance nominally is already contained in the concept of that substance, then possibility has to be added to that complete concept.

- Note: Leibniz do not make a distinction between objects and language.

Example

- That Caesar would have crossed the Robicon contradicts with that Caesar would have not crossed the Robicon when the two sentences are actualized at the same time, so the contradictory modal predicates (would have crossed the Robicon or not) partition the complete concept (Caesar) into different and disjoint sub-concepts. Then there are many Caesars to satisfy the conditions of these sub-concepts.

Argument

- (1) A complete concept includes all and only the properties by which a unique individual substance can be specified.
- (2)According to (1), a complete concept 'one to one' corresponds to an individual substance.
- (3) To avoid Spinoza's necessitarianism, a complete concept also includes modal properties.
- (4) Modal properties contradict with each other and can not co-exist when they are both realized at the same time.
- (5) According to the principle of contradiction, contradictory modal properties can not be realized in the same complete individual concept.
- (6) According to (3) and (4), there are many individuals correspond to the complete concept.
- (7) Therefore, in order to avoid Spinoza's necessitarianism, (2) is false, but (6) is true.

- Therefore, the effort to establish the complete concept and to close the domain of that concept to specify a unique individual substance unfortunately leads to the negative result: many possible individuals have been introduced in the complete concept.

Ontological Status of Possible Individual

- Whereas the predicates have partitioned the complete concept into different sub-concepts, each sub-concept can be partitioned further. Because of these infinite regress, each sub-concept can not correspond to a unique individual, and there are forever several possible individuals fitting the corresponding sub-concept. As a result, there is no ultimate reality for possible individuals to inhabit, and possible individuals do not really exist.

- The above process is analogue to the divisibility of material body. The point that I insist on is to analyze Leibniz's possible worlds in terms of monadology, his mature system.

3. Compossibility

- The relation of compossibility is related with the question of how to form a possible world by possible individuals.

Debate

- There are debates over compossibility among scholars:

- (1) whether compossibility should be interpreted analytically or synthetically.

- (2) whether the relationship of compossibility is transitive or not.

3.1 Analytically or Synthetically

- Some scholars think that two possible things are incompossible if the actualization of both leads to a logical contradiction.

- Others think that compossibility means the orderliness and lawfulness of relations among substances.

My View

- There is a significant change in Leibniz's thought, which can explain the difference.
- Because it is hard to explain the ultimate reality of possible individuals, Leibniz's interest changed from complete concepts to the spontaneous principle of action (εντελεχη), that is, a shift from the traditional logic and metaphysics to the modern dynamics.
- Because the static analysis of possible individuals does not reveal the ultimate elements that constitute the complete concepts, Leibniz resorted to active force.

Quotation

- *On the Correction of Metaphysics And the Concept of Substance*:

- 'the concept of forces or powers, which the Germans call Kraft and the French la force, and for whose explanation I have set up a distinct science of dynamics, brings the strongest light to bear upon our understanding of the true concept of substance. Active force differs from the mere power familiar to the Schools, for the active or faculty of the Scholastics is nothing but <u>a close possibility of acting</u>, which needs an external excitation or a stimulus, as it were, to be transferred into action. Active force, in contrast, contains <u>a certain act or entelechy</u> and is thus midway between the faculty of acting and the act itself and involves <u>a conatus</u>.'

My View

- From the logical perspective, a complete concept includes all and only the properties (or predicates) by which a unique individual can be specified, but shows nothing about the interrelations between one individual concept and another (let us assume that all the predicates contained in the complete concept are monadic predicates).

- From the dynamical perspective, possible individuals are not absolutely independent from each other, but have an internal and combinatorial structure through which they try to cooperate with each other to realize or fulfill themselves.

- So the above difference between the logical compossibility and orderly compossibility can be bridged by the development of Leibniz's thought.

3.2 Transitive or not

- Benson Mates has argued that the relation of compossibility between individual concepts is an equivalence relation, because it is reflective, symmetric and transitive.

- Fitch criticized Mates' view and said that 'it is surely possible that all the members of a given class be pair-wise compossible but the class itself not be compossible.'

Mondadori's View

- 'The question has often been raised of whether or not Leibniz took the relation of compossibility to be an equivalence relation, or whether or not his logical calculi and his metaphysical system require that the relation of compossibility be reflective, symmetric, and transitive. The negative answer is indeed correct, provided we take the relata of that relation to be incomplete concepts. But an affirmative answer is also correct, provided we take the relata of that relation to be complete individual concepts, and provided each of the constituents of any given possible world w is taken to mirror, to be mirrored by, and to be connected with, every other constituents of w. '

The 'Mirroring' Argument

- (1) a monad is living in a world iff that monad is a mirror of the whole world.
- (2) a monad is a mirror of the whole world iff each monad is a mirror of all other monads living in the same world.
- (3) a monad is a mirror of all other monads living in the same world iff each monad is mirroring and mirrored by all the other monads.
- (4) according to (1)(2)(3), two monads are living in the same world iff one is mirroring and mirrored by the other.
- (5) two monads are compossible with each other iff they are living in the same world.
- (6) according to (5), two monads are compossible with each other iff one is mirroring and mirrored by the other.
- (7) therefore, the relation of compossibility is transitive iff the relation of mirroring is transitive.

A Vicious Circle?

- If the relation of mirroring is transitive, then the relation of compossibility is also transitive.
- However, the transitive relation of mirroring has already presupposed that monads which are mirroring each other are living in the same world.
- In my view, this difficulty is related to Leibniz's doctrine of universal connection and his alleged reduction of relation.
- According to Leibniz, each binary relation or n-ary relation can be extended to or has already presupposed an infinitary relation, that is, Leibniz's doctrine of universal connection. And each infinitary relation can be reduced to an unary predicate, that is, a monad's mirroring of the whole world.

Reducibility of Relations

- As is well known, Leibniz denies that relations are real, because any relation can be reduced to non-relation, that is, the subject-predicate form.

For example:

(1) Theaetetus is taller than Socrates.

which can be supported by two other propositions:

(2) Theaetetus is 6 feet tall.

(3) Socrates is 5 feet tall.

but, to asserted (1), we still need the following:

(4) 6 feet > 5 feet.

My View

- The difference between language and metaphysics:
- If we regard the possible individuals as objects, that is, the ultimate reality can be arrived at the end of the divisibility, then a possible world is an equivalence class.
- If we do not regard the possible individuals as objects, that is, there is no ultimate reality because the modal division is infinite, then a possible world is a maximal consistent set.

1.4 The Principle of the Best

- Simplicity and variety are two criteria by which God choose among all the possible worlds.

- 'It follows from the supreme perfection of God that he has chosen the best possible plan in producing the universe, a plan which combines the greatest variety together with the greatest order; with situation, place, and time arranged in the best way possible; with the greatest effect produced by the simplest means; with the most power, the most knowledge, the greatest happiness and goodness in created things which the universe could allow.'

Debate

- Like other topics we have discussed in the above, there are also debates about Leibniz's doctrine of the best possible world.

Rescher's View

- Rescher has said: the two real variables are simplicity and variety, and their relation is not cooperative but competitive: 'too simple laws produce monotony; too varied phenomenon produce chaos.'

- 'Determining the maximum or minimum of that surface-defining equation which represents a function of two real variables specifically requires those problem-solving devices for which the mechanisms of the differential calculus were specifically devised.'

The Device of Differential Calculus
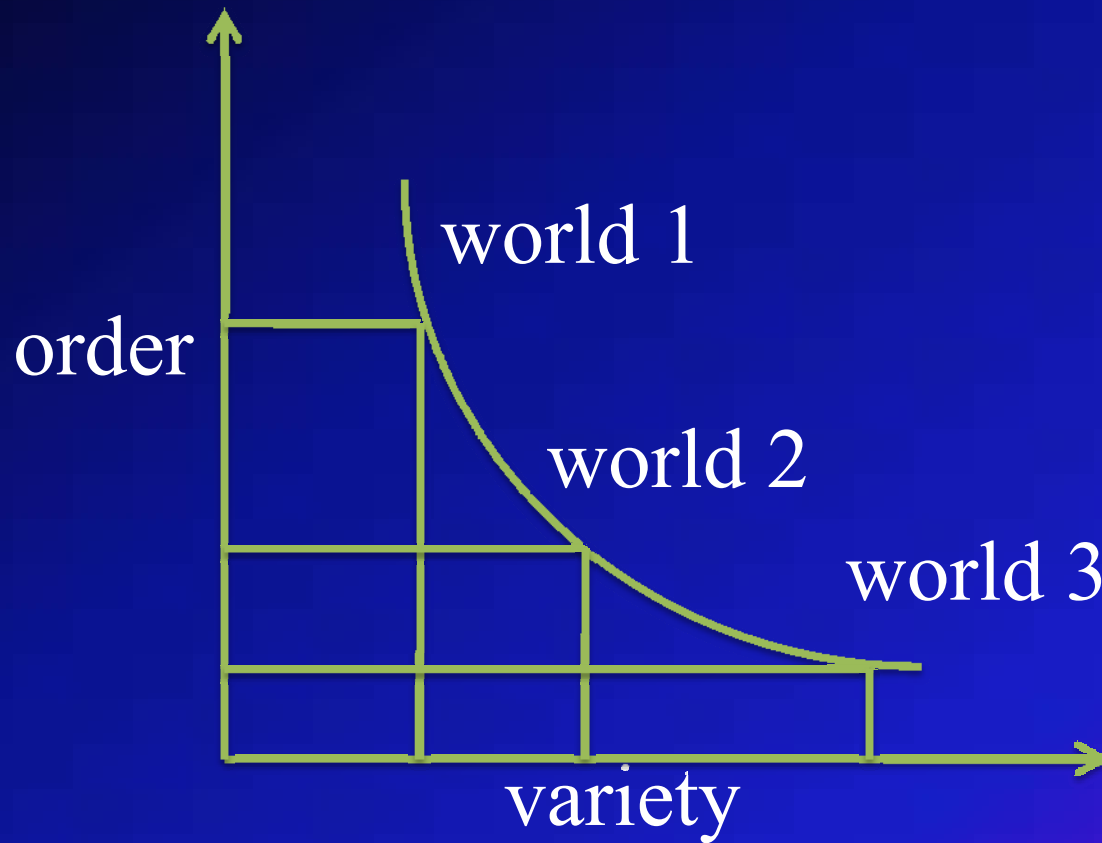
Blumenfeld's View

- Blumenfeld thought that simpler does not mean fewer, 'since exceptions make a law more complex, a necessary condition for maximum simplicity, is that the law be strictly universal, or exception-free.'

- Therefore, 'the mutual accommodation of the infinite multiplicity of simple substance is the means of obtaining the greatest variety possible, but with the greatest possible order. The harmony above rests on the harmony below.'

My View

- If the ultimate reality has been achieved as a metaphysical hypothesis, then lawfulness and orderliness have been set up in the internal structure of possible individuals.

- So the true simples lay foundations for the worlds' lawfulness and orderliness, and the 'well founded phenomena' are thus founded on these simples.

- Therefore, the simplicity is the means to achieve the variety.

My View

- If the ultimate reality can not be achieved, then any simple can be divided further into more simple ones, and we can not reach the bottom of the bottomless chasm of the simplest.

- Therefore, the further we go along the abyss, the more confused we become, and we should keep the balance between the simplicity and variety.

1.5 Summary

- There are three stages in our analysis:
- (1) It is the individual substance in our common sense that we attempt to give a complete concept;
- (2) The complete concept has to be divided into possible individuals, as we talk about what might have happened to that individual substance in our common sense;
- (3) At the end of the division, we arrived at the true atoms (monads), and these monads are the true substance in the philosophical sense.

The Focus of Debates

- In each step, we have also explained some debates among scholars. In my view, the focus of all the debates about Leibniz's doctrine of possible worlds lies in whether Leibniz's ontology has reasonably established the foundation of ultimate reality, that is, whether monads—the true atoms, the souls, and the active force—have been fully described by the complete concepts, or are just a metaphysical hypothesis.

Three Different Ways

- Because of Leibniz's scattered remarks and the inconsistency between his esoteric and exoteric doctrines, there may be three ways to interpret the ultimate reality as the result of infinite division of individual concepts:
- (1) the division can not be terminated, and we can not get to the true atoms except purely possible individual which can be divided further;
- (2) the division can be terminated, and we can get to the ultimate reality as individual substances, whether that is guaranteed by metaphysical hypothesis or dynamical laws;
- (3) the division can be terminated but the terminated possible individuals do not really exist——they are only exist in the God's mind (or human's minds).

Further Questions

- In my view, there are closely interrelated connections between Leibniz's monadological metaphysics and his differential and integral calculus, that is, between the true atoms and the infinitesimals.

- Therefore, the following three concepts are interrelated:

    (1) true atoms

    ——the division of material body.

    (2) possible individuals

    ——the division of an individual concept

    (3) infinitesimals

    ——the division of real number.

6. Two Formal Systems

- 2.1. Mates' System
- 2.2. Fitch's System

6.1. Mates' System

- All predicates other than the identity predicate are of rank 1

- A complete individual concept is a set of simple properties satisfiable by exactly one thing and containing all the simple properties that would belong to that one thing if it existed.

- Compossibility is an equivalence relation in the set of all complete individual concepts, partitioning it into equivalence classes, called possible worlds.

Nonlogical Constants

- (1) The set of individual constants is mapped onto the set of complete individual concepts;

- If β is an individual constant, let $C(\beta)$ be the complete individual concept associated with β.

- (2) The set of singulary predicates is mapped onto the set of simple properties;

- If θ is a singulary predicate, let $C(\theta)$ be the simple property associated with θ.

Symbols

- Let W a possible world,
  - φ, ψ, χ formulas,
  - α a variable,
  - β, γ individual constants,
  - θ a predicate other than the identity predicate.

Semantics

- If φ is θβ, then φ is true of W iff C(θ) ∈ C(β) and C(β) ∈ W
- If φ is β=γ, then φ is true of W iff C(β) = C(γ) and C(β) ∈ W
- If φ is ¬ψ, then φ is true of W iff ψ is not true of W
- If φ is (ψ→χ), then φ is true of W iff either ψ is not true of W or χ is true of W
- If φ is ∀αψ, then φ is true of W iff ψα/β is true of W for every individual constant β such that C(β) ∈ W
- If φ is □ψ, then φ is true of W iff ψ is true of every possible world W′.

6.2. Fitch's System

- A concept is a set of properties.
- A complete individual concept (cic) is a set of properties such that it is possible that there is one object that has all and only those properties in the set.

Postulates

- a set K is compossible if and only if K is a nonempty set of cic, and it is possible that for any cic X, if $X \in K$, then X is realized.

- a set K is maximal if and only if for any cic X, if $X \notin K$, then $K \cup \{X\}$ is not compossible.

- a set W is a possible world if and only if W is maximal and W is compossible.

- a cic C reflects a cic D if and only if it is not possible that C is realized and D is not realized

- a cic C mirrors a possible world W if and only if for any cic D, if $D \in W$, then C reflects D.

- a set K involves a cic C if and only if K is a nonempty set of cic and it is not possible that for any X, if $X \in K$ then X is realized and C is not realized.

- a set K is closed if and only if K is a nonempty set of cic and for any cic C, if K involves C, then C is a member of K

Axioms

- Every cic is a member of some possible world.
- For every possible world W and for any cic C, if C is a member of W, then C mirrors W.

Language

- individual constants: $a_1, a_2, a_3, \cdots$
- individual variables: $x_1, x_2, x_3, \cdots$
- (unary) predicates: $F_1, F_2, F_3, \cdots$, identity: $=$
- logical signs: $\neg, \vee, \forall, \square$

Interpretation

- Interpretation: ⟨D, G, W, V, f, K⟩
- D = the set of all complete individual concepts
- W = the set of all possible worlds
- G = the set of all properties
- f is a function such that it assigns to each constant in the language a complete individual concept (from D) and to each (unary) predicate a property (from G)

- K is the counterpart relation such that:
- $\forall a(a \in D \rightarrow Kaa)$
- $\forall a \forall b(Kab \rightarrow a \in D \wedge b \in D)$
- $\forall w \forall a((w \in W \wedge a \in w) \rightarrow \forall b(b \in w \rightarrow (Kba \rightarrow a = b)))$
- $\forall w_1 \forall w_2 \forall a \forall b \forall c((w_1 \in W \wedge w_2 \in W \wedge a \in w_1 \wedge b \in w_2 \wedge c \in w_2) \rightarrow ((Kba \wedge Kca) \rightarrow b = c))$
- $\forall w_1 \forall w_2 \forall a \forall b \forall c((w_1 \in W \wedge w_2 \in W \wedge a \in w_1 \wedge b \in w_2 \wedge c \in w_2) \rightarrow ((Kab \wedge Kac) \rightarrow b = c))$

- V is a valuation function such that:
- If φ is Fa, then V(φ, w_i)=T iff $f(a) \in f$(F)
- If φ is ¬ψ, then V(φ, w_i)=T iff V(ψ, w_i)≠T
- If φ is χ∨ψ, then V(φ, w_i)=T iff either V(χ, w_i)=T or V(ψ, w_i)=T
- If φ is $\forall x$ψ, then V(φ, w_i)=T iff for every a such that $f(a) \in w_i$, then V(ψx/a, w_i)=T
- If φ is $a=b$, then V(φ, w_i)=T iff $f(a)$ is $f(b)$

Thank you!